

# Determining the seismic age of the young open cluster $\alpha$ Per

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## ABSTRACT

In this work we aim at constraining the age of the young open cluster Melotte 20, known as  $\alpha$  Per, using seismic indices. The method consists of the following steps: 1) Extract the frequency content of a sample of stars in the field of an open cluster. 2) Search for possible regularities in the frequency spectra of  $\delta$  Sct stars candidates, using different techniques, such as the Fourier transform, the autocorrelation diagram, the frequency difference histogram and the échelle diagram. 3) Constrain the age of the selected stars by both the physical parameters and seismic indices by comparing against a grid of asteroseismic models representative of  $\delta$  Sct stars. 4) Find possible common ages between these stars to determine the age of the cluster. We performed the pulsation analysis with MULTIMODES, a rapid, accurate and powerful open-source code, which is presented in this paper. It has been tested with a sample of 32 stars observed by *TESS* in the field of  $\alpha$  Per, from which we have found nine new  $\delta$  Sct stars which raise the total number of  $\delta$  Sct stars to 11 for the cluster. We have obtained the low-order large separation in four of them: TIC 410732825, TIC 354792288, TIC 285935852, and TIC 252829836. Two of these four stars, TIC 410732825 and TIC 285935852, show a splitting of frequencies in their frequency difference histogram, resulting in a possible angular rotation frequency of around 10  $\mu$ Hz. Considering that they are ZAMS stars, we have used the corresponding relation between the frequency at maximum power and effective temperature to constrain the age of  $\alpha$  Per between 103 and 131 Myr. This is an improvement over different techniques in the past. We therefore show that space asteroseismology is capable of taking important steps in the dating of young open clusters.

**Key words:** asteroseismology – delta Scuti stars

## 1 Introduction

Open clusters are laboratories of stellar and galactic astrophysics. Having at our disposal a group of tens of stars of similar chemistry and ages, within a narrow field of observation, allows us to better constrain models with which we can compare our observations. We can better characterize the stars in this way, determining their evolutionary stages and, by extension, the origin and evolution of the galaxy which they belong to.

There are many sources that can contribute to ambiguity determining the age of a cluster by isochrone fitting on the Hertzsprung-Russell (HR) diagram: uncertain distance modulus, cluster membership, binarity of individual stars, reddening and extinction, metallicity, different treatments of the physics of the models used to derive the theoretical isochrones, and the uncertainties from isochrone fitting methods. Ambiguity is even greater in young clusters, with a sparse population of stars leaving the main sequence (turn off). The tip of the red giant branch is used to date globular clusters but cannot be used in young open clusters.

Pulsating stars can allow a better characterization of the cluster, because some of their seismic indices are directly related to internal characteristics of the stars that we want to date. Some types of classical pulsating stars, such as  $\delta$  Sct stars, have a rich and varied frequency spectra. They correspond to A and F stars of intermedi-

ate mass, very common in the main sequence (MS) for young open clusters. Space missions such as CoRoT (Baglin et al. 2006), *Kepler* (Koch et al. 2010) and *TESS* (Ricker et al. 2014) have lowered the detection threshold for their signals. Being grouped around the MS, it is not easy to estimate the age by isochrone fitting. In addition, it has been very difficult, until recently, to find regularities in the frequency spectra of these stars, as in the case of solar-type. There are studies showing that it is possible to find regularities in the frequency pattern of  $\delta$  Sct stars, even out of the asymptotic regime (see for example, García Hernández et al. 2009; Paparó et al. 2016; Barceló Forteza et al. 2017; Bedding et al. 2020, and references therein). This is the named large separation, like for solar-type stars, but in the low-order regime (from  $n=2$  to  $n=8$ , Suárez et al. 2014). It is defined as the difference between modes of the same degree and consecutive orders:

$$\Delta \nu_{low} = \nu_{n,l} - \nu_{n,l-1} \quad (1)$$

We used here the subscript *low* to differentiate it from that of the asymptotic regime. However, as for solar-type stars, this  $\Delta \nu_{low}$  is also related to the stellar mean density (Suárez et al. 2014; García Hernández et al. 2015, 2017).

Other seismic indices are the rotational splitting of modes of the same order and degree, related to the angular rotation, and the frequency at maximum power, used in solar-type stars, directly related to the effective temperature, that has been also found in  $\delta$  Sct stars

(Barceló Forteza et al. 2018, 2020; Bowman & Kurtz 2018; Hasan-zadeh et al. 2021, BF2018, BF2020, BK2018 and H2021 from now on).

Here we use the asteroseismic indices to study the age of the open cluster Melotte 20 (also known as  $\alpha$  Per). The structure of the paper is as follows: in Sec. 2 we provide the main characteristics of the cluster and former estimates of its age. In section Sec. 3 we describe the sample of stars used in this research. In Sec. 4 we introduce the code MULTIMODES, a new tool for the analysis of pulsating stars. In order to show its reliability, we have compared it to one of the most reliable and most used in the field, SIGSPEC (Reegen 2007). We explain the details of the code in this section, and also the comparative results obtained between them, using synthetic and real light curves, in terms of accuracy and computing time. In this section we also present the list of  $\delta$  Sct stars candidates identified in our sample with the frequency analysis. In Sec. 5 we show how, in four of them, it has been possible to measure the low-order large separation. The angular rotation and the frequency at maximum power of two of these stars have been measured. In Sec. 6 we explain the details of the grid of pulsation models calculated with MESA (Paxton 2019) and FILOU (Suárez & Goupil 2008), in order to constrain the models that better fit to the seismic observations. In Sec. 7 we compare our asteroseismic age with previous works and discuss the reliability of the proposed method to date open clusters. Finally, in Sec. 8, the main conclusions are exposed.

## 2 $\alpha$ Per

The open cluster  $\alpha$  Per is located in the constellation of Perseus, at a distance of  $174.89 \pm 0.16$  pc, obtained from *Gaia* DR2 parallaxes (Gaia Collaboration et al. 2018). In Lodieu et al. (2019) a total of 517 astrometric member candidates had been identified in the tidal radius of  $\alpha$  Per, using a kinematic method combined with the statistical treatment of parallaxes and proper motions. Netopil & Paunzen (2013) estimated solar metallicity for all members of the cluster and an extinction along the line of sight of around  $A_V = 0.3$  (Prosser 1992).

Recent age estimates by isochrone fitting range from 20 to 90 Myr. Makarov (2006) estimates the age in around 52 Myr, fitting isochrones in the  $M_V$  vs. (B-V) diagram, with  $Z = 0.02$ ,  $E(B-V) = 0.055$ , and overshooting computed from the models by Pietrinferni et al. (2004). According to Silaj & Landstreet (2014), the age of  $\alpha$  Per is  $60 \pm 7$  Myr, determined with isochrones by Girardi et al. (2000), with  $Z = 0.02$ , fitted onto an HR diagram, considering that the uncertainties are less ambiguous than using a colour-magnitude diagram. Isochrones are most constrained around the one that passes through the brightest main sequence star,  $\psi$  Per (HD 22192), which lies close to the terminal age main sequence (TAMS). In both works, the estimated age of the cluster relies on the parameters of a single star, with a difference of around 8 Myr in their results. Such an approach must be taken with caution, since errors in the estimate of luminosities and temperatures for stars in different evolutionary stages can be quite large. These errors may come from, e.g. the type of photometry to use bolometric corrections and the correct modulus distance, or even the lack of corrections for rotation effects (see e.g. Suárez et al. 2002, , for more details).

Other works use spectroscopic observations of lithium in low-mass stars to estimate the age of young clusters (see for example Basri & Martín (1999), Stauffer et al. (1999)). The idea is that for stars near the substellar mass limit, the age is very sensitive to the lithium depletion boundary, when they start to burn lithium in their cores. As these stars are fully convective in their range of masses,

the abundance of lithium is visible in their atmospheres through the 6708 Å Li I doublet. The problem with this method is that unresolved binaries makes the cluster appear younger, as claimed by Martín et al. (2001). For this reason, they used the faintest members to derive the age of the cluster, for which lithium has not been detected. Another difficulty with this method is its dependency on good models that take into account the characteristics of the stellar atmosphere. In Stauffer et al. (1999) the estimated age with this method is around  $90 \pm 10$  Myr, noticeably older than Makarov and Silaj & Landstreet.

Lodieu et al. assumes an age of 90 Myr to compile the census of stars that are members of the cluster, because it is the most common age in the literature.

## 3 Our sample

Firstly, we have done a cross match between VizieR Online Data Catalogue *Gaia* DR1 open cluster members (Gaia Collaboration et al. 2017) and TESS Input Catalogue (TIC) (Stassun et al. 2019), searching possible targets belonging to the same open cluster, from which we could obtain light curves from TESS mission. We have found a list of 112 stars in the field of  $\alpha$  Per, with measured values for parallax, mean G magnitude,  $E(B - V)$  reddening, bolometric corrections estimated by Andrae et al. (2018), and effective temperature. They all contained in the census prepared by Lodieu et al., obtained from Gaia Collaboration et al. (2018). We have estimated the G magnitude with equation:

$$M_G = G - 5 \log r + 5 - A_G, \quad (2)$$

where distance  $r$  is taken as the inverse of the *Gaia* parallax. Extinction  $A_G$  is calculated from reddening values as in Stassun et al.:

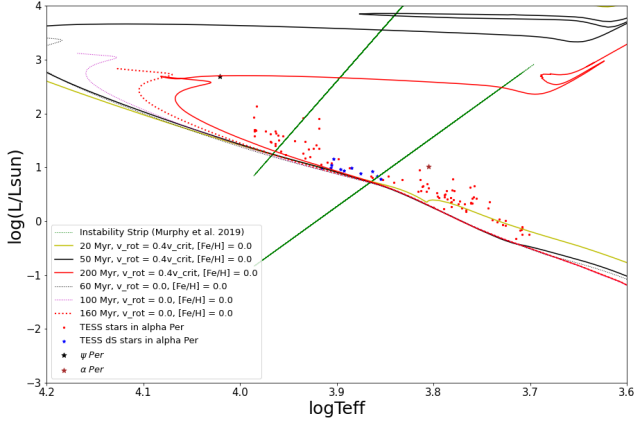
$$A_G = 2.72 E(B - V). \quad (3)$$

The G magnitude is converted to luminosity using the bolometric correction  $BC(T_{\text{eff}})$  with the equation:

$$-2.5 \log L = M_G + BC_G(T_{\text{eff}}) - M_{\text{bol}\odot} \quad (4)$$

Figure 1 depicts the HR diagram of our sample, including MIST\* isochrones (Dotter 2016; Choi et al. 2016; Paxton et al. 2011, 2013, 2015), with solar metallicity, of different ages and rotational velocities. They all have been computed from the pre-main sequence phase (PMS) to the end of hydrogen burning. In rotating models, solid-body rotation with  $\Omega/\Omega_{\text{crit}} = 0.4$  is initialized at zero age mean sequence (ZAMS). Rotation makes the star hotter or cooler depending on the efficiency of rotational mixing in the envelope. If rotational mixing introduces a sufficient amount of helium into the envelope, increasing the mean molecular weight, the star becomes more compact and hotter. On the contrary, if rotational mixing is not efficient, the centrifugal effect dominates, making the star cooler and more extended. The ages of the isochrones are taken to cover the wide range of values found in the literature (see previous section). We can see that all the calculated isochrones between 20 and 200 Myr are quite overlapping in the MS zone. We also have located the position of  $\psi$  Per on the HR diagram, which is compatible with an age of 200 Myr, quite far from the age estimated by Silaj & Landstreet. Following Makarov (2006), we used the *Hipparcos* re-reduction parallax, and the mean reddening of the entire sample, since there is no *Gaia* parallax nor TIC reddening for this star. It is located quite far from the isochrones drawn on our diagram. Moreover, although most of the stars of the

\* <http://waps.cfa.harvard.edu/MIST/>



**Figure 1.** HR for the sample of 112 stars from the field of  $\alpha$  Per, taken from TESS Input Catalogue. Isochrones between 20 and 200 Myr with solar metallicity are taken from MIST. (<http://waps.cfa.harvard.edu/MIST/>). The borders of instability strip are also drawn, calculated according to Murphy et al. (2019). The position of  $\psi$  Per and  $\alpha$  Perseus are also indicated, two of the most luminous stars in the cluster

sample lie well inside the instability strip (Murphy et al. 2019),  $\alpha$  Per remains out of its red edge, which might be due to a wrong estimate of its physical parameters.

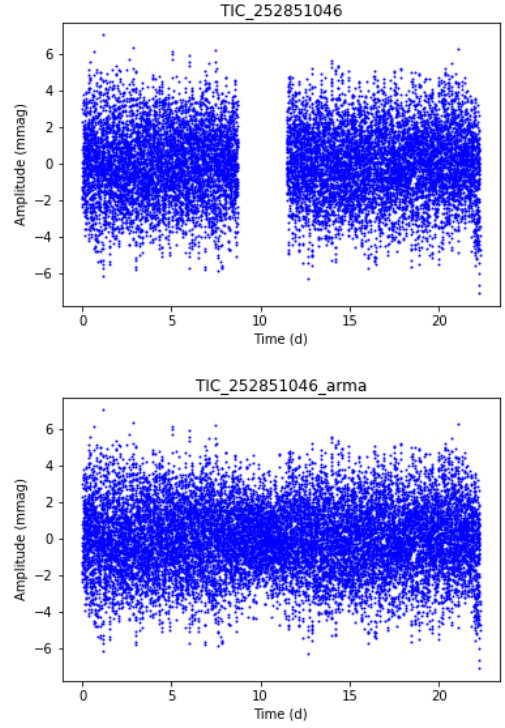
This is where asteroseismology plays an important role, since it provides independent seismic indices that provide accurate determination of global stellar parameters, including the age of the star.

We analyzed a set of 32 stars using data from sector 18 of the *TESS* mission, with approximately 14700 points, a Rayleigh resolution of approximately  $0.045 d^{-1}$  and a cadence of two minutes. We used the Pre-Search Data Conditioned (PDC) light curves, corrected for instrumental effects, that are publicly available through the MAST<sup>†</sup>. These light curves have a 6.2 hours gap, between TJD 1791.1115 and TJD 1791.3699, caused by the satellite downlink, when the spacecraft passed through the shadow of the Earth at start of orbit 43 (Fig. 2). Pascual-Granado et al. (2018) showed that the analysis of light curves with gaps can produce an undesirable amount of spurious peaks. Following Pascual-Granado et al. (2015) we interpolated our data using the MIARMA code, which uses autoregressive and moving average models. This method is aimed to preserve the original frequency content of the light curve, making the frequency extraction more reliable than leaving the gaps or using other gap-filling methods.

## 4 Frequency analysis

### 4.1 MultiModes: a new tool for analysis of pulsating stars

Many of the light curves provided by space missions are uniformly sampled, but show some gaps, due to lack of observations, instrumental issues or environmental effects. The Lomb Scargle Periodogram (from now on LS) is very powerful for analyzing non-uniformly sampled time series (Scargle 1982). For uniform sampling, LS provides the classical periodogram. The algorithm calculates, at each frequency of interest, a time phase shift, and which in turn is used to evaluate the power spectra at that frequency. This time phase shift makes the LS to be independent of shifting all the points by any con-



**Figure 2.** Top: Light curve of TIC 252851046. Bottom: Interpolated light curve of TIC 25851046 with MIARMA algorithm

stant. In this way, the calculation of LS is equivalent to performing a least squares fit of data to a sinusoidal function for each evaluated frequency. So there is a deep connection between the Fourier Transform and least squares analysis in the LS.

Computing LS can be very slow because it requires a number of calculations of the order of  $N^2$ ,  $N$  being the number of points in the sample. For this reason, it is implemented the Fast Lomb Scargle Evaluation (Press & Rybicki 1989), based on extrapolation from unevenly sampled data. The extrapolation consists of substituting the values of the series for others constructed by means of an evenly sampled mesh, so that the trigonometric sums that require the calculation of the periodogram approximate those of the original values. On this basis, the order of the calculations for the evaluation of the periodogram is reduced to  $N \log N$ .

The width of a peak does not depend on the number of sampled points or the signal to noise ratio (SNR), it only depends on Rayleigh resolution, which is inversely proportional to the total sample size (VanderPlas 2018). Two peaks within the Rayleigh resolution, in theory, could not be perfectly resolved by the periodogram. However, the stochastic nature of the series can cause numerous spurious frequencies to emerge, many of them of low amplitude but above the mean noise level. As detailed in Balona (2014), the problem of using the pre-whitening technique lies in the large number of artificial frequencies that are generated with a high SNR. When, at each step, a signal very similar to the one to be extracted is added to the analyzed light curve, if the fit has not enough accuracy, it can generate an interference pattern around the added frequency.

These spurious frequencies can be avoided, to a great extent, by making a simultaneous fit to all the extracted frequencies, through non-linear optimization. We use least squares fit to a multisine func-

<sup>†</sup> <https://archive.stsci.edu/>

tion, taking as parameters the frequencies, amplitudes and phases of each of the peaks to be extracted.

The well-known SIGSPEC algorithm (Reegen 2007, SS from now on) uses pre-whitening to extract all those frequencies that are above a certain criterion of significance, defined from the False Alarm Probability (FAP) as:

$$\text{sig} = -\log(\text{FAP}) \quad (5)$$

taking into account dependency of FAP with the phases.

Widely used in asteroseismology, it is not open source, and no longer updated.

Inspired by SIGSPEC methodology, we developed MULTIMODES(MM from now on), which is a Python routine developed to extract the most significant peaks of a sample of classical pulsating stars. It is available in a public Github repository<sup>‡</sup> and it is customizable. This routine has been designed using Astropy package<sup>§</sup> to calculate the periodograms and the LMFIT package<sup>¶</sup> for non-linear optimization of the extracted signals.

The algorithm fits frequency, amplitude and phase through non-linear optimization, using a multisine function. This function is re-defined with the new calculated parameters. It does a simultaneous fit of a number of sinusoidal components (20 are usually enough).

Then they are subtracted from the original signal and goes back to the beginning of the loop with the residuals as input, repeating the same process, until the stop criterion is reached. After that, the code can filter suspicious spurious frequencies, those of low amplitude above the Rayleigh resolution, and possible combinations of modes.

#### 4.2 Accuracy test with simulated spectra

We tested the reliability of MM using synthetic light curves. Following Balona (2014), we constructed artificial light curves with 50, 100, 200 and 400 frequencies, with uniformly distributed values between 0 and  $30 \text{ d}^{-1}$ , amplitudes ranging between 0 and 10 mmag, exponentially distributed towards low values, phases uniformly distributed, and adding a Gaussian noise of about 0.5 mmag.

The frequency deviation of MM is similar to that of SS in the curves with 50 and 100 frequencies, around  $10^{-4} \text{ d}^{-1}$ , when we used an unlimited number of components for the simultaneous fit of the simulated signal (Fig. 4). With a higher number of frequencies, 200 and 400, SS is more accurate than MM by one order of magnitude, because MM used a limited number of components for the simultaneous fit, no more than 50. Still, results with MM have a enough accuracy for a precise frequency extraction. As the frequency density increases, the problem becomes more unstable since more spurious frequencies appear, with both MM and SS. Therefore, it makes sense to limit the number of components of the simultaneous fit of the light curve when frequency density is very high. In this sense, MM allows us to have greater control over the frequency analysis, by limiting the number of components for the fit, and working with frequency packs.

#### 4.3 Frequency content of the $\delta$ Scuti sample

From the sample of 32 analysed stars analyzed with MM, we found that 11 of them are  $\delta$  Sct stars stars, or hybrids, a type of pulsating stars with intermediate mass that show low-order acoustic oscillations (p-modes) and also high order gravity modes (g-modes),

<sup>‡</sup> <https://github.com/davidpamos/MultiModes>

<sup>§</sup> <https://www.astropy.org>

<sup>¶</sup> <https://lmfit.github.io/lmfit-py/>

**Table 1.** Comparative analysis between SS and MM extracted peaks, using ARMA-interpolated light curves with MIARMA code, for the 11  $\delta$  Sct stars sample from  $\alpha$  Per.  $N_{\text{SSar}}$  and  $N_{\text{MMar}}$  are, respectively, the number of extracted frequencies by SIGSPEC and MULTIMODES. SSarMMar is the degree of coincidence between SIGSPEC and MULTIMODES

TIC	$N_{\text{SSar}}$	$N_{\text{MMar}}$	SSarMMar
104319359	249	238	97.9
116011834	233	222	97.7
252829836	82	76	97.4
252851046	213	222	93.7
285935852	45	40	97.5
347570557	189	188	94.7
354638295	97	89	98.9
354792288	55	47	100.0
401079326	39	35	100.0
410732825	69	55	100.0
428320122	71	64	100.0

more typical of  $\gamma$  Dor stars (Grigahcène et al. 2010; Uytterhoeven et al. 2011). Except TIC 252829836 and TIC 347570557, 9 stars in our sample have not been identified as  $\delta$  Sct stars so far.

We have analysed the frequency content of these 11 stars using MM and SS, for greater reliability. The frequency comparison assumes that both codes identify the same frequency when their difference is smaller than the Rayleigh resolution. We established the degree or percentage of coincidence based on whether each frequency extracted by MM is among those extracted by SS or not. The degree of coincidence between MM and SS in the values of the extracted frequencies, extracting the most significant frequencies, is above 90% in all the cases (Tab. 1). We have also done a comparative analysis on the computing speed between MM and SS with our sample of 11  $\delta$  Sct stars stars (see Ap. A).

## 5 Seismic indices of $\delta$ Scuti stars

Among our sample of 11  $\delta$  Sct stars, in four of them, TIC 410732825, TIC 354792288, TIC 285935852 and TIC 252829836 (Fig. 5), we found a pattern that we identify as a low-order large separation ( $\Delta\nu_{\text{low}}$ ), following García Hernández et al. (2009) and Ramón-Ballesta et al. (2021) techniques. The autocorrelation function (AC), the Fourier transform (FT), the histogram of differences of frequencies (FDH), and the échelle diagram (ED) were applied to the 30 frequencies with highest amplitudes and above  $5 \text{ d}^{-1}$  to avoid

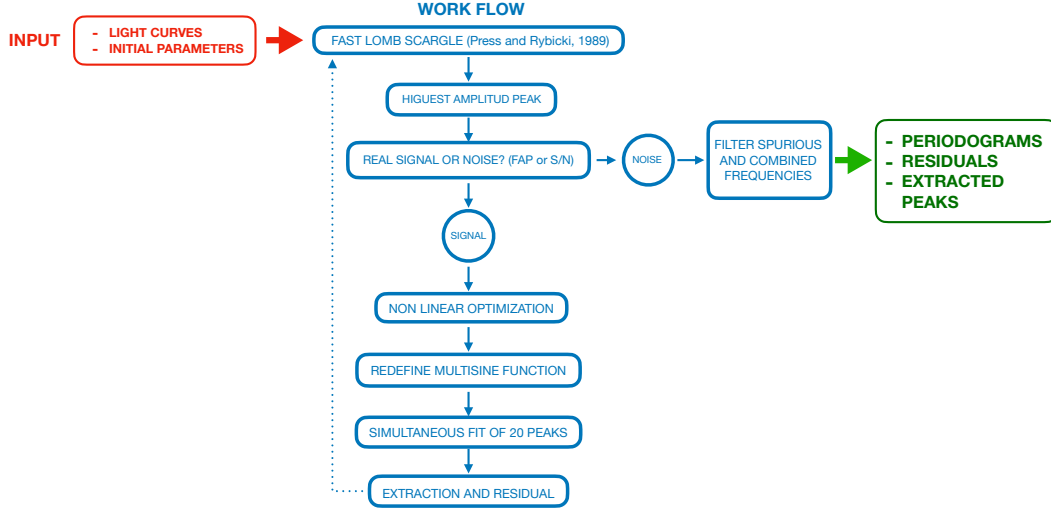


Figure 3. Workflow of the MultiModes algorithm

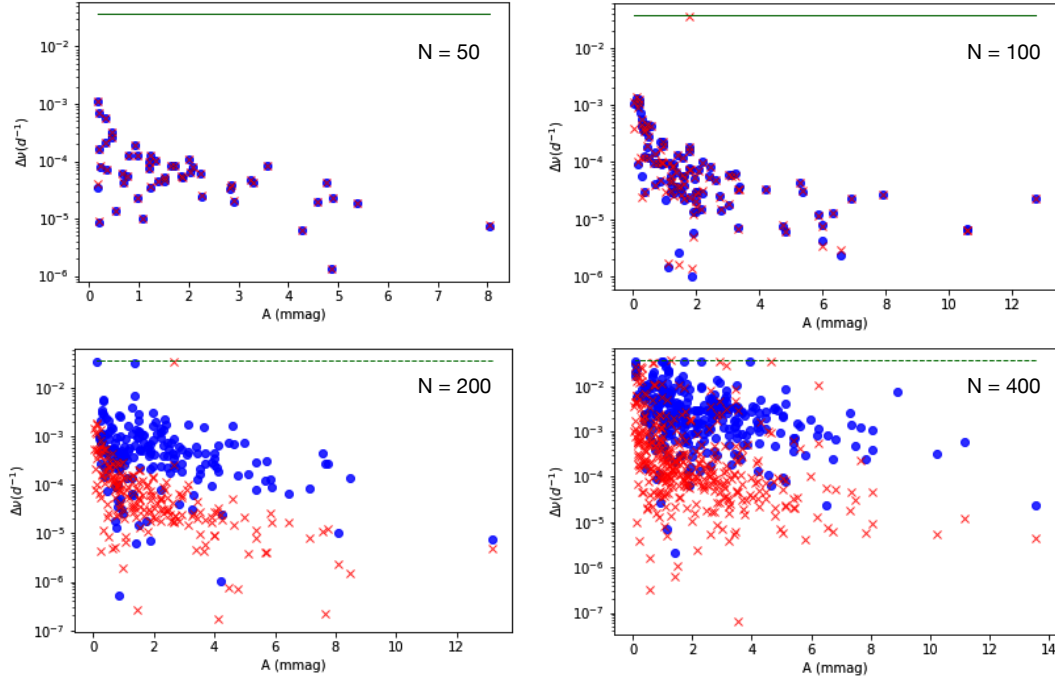


Figure 4. Comparative analysis of accuracy between MM and SS, using simulated light curves with different number of frequencies, 50 (upper left panel), 100 (upper right panel), 200 (bottom left panel) and 400 (bottom right panel). Each plot represents the frequency deviation of the extracted peaks with MM (blue filled circles) and SS (red crosses). The dotted green line is the level of the Rayleigh resolution.

g modes. These frequencies are selected by amplitude but they are given equal amplitudes when computing the transformations.

Fig. 6 and Fig. 7 show, respectively, the cases of TIC 354792288 and TIC 410732825 (see the other two stars in Ap. B). For TIC 354792288, the FT, AC and FDH show a peak around  $42 \mu\text{Hz}$ . It may be half the value of the low-order large separation. The AC and FDH also show a peak around  $84 \mu\text{Hz}$ . The ED shows the alignment of several frequencies when  $86 \mu\text{Hz}$  is chosen as the low-order large separation. Thus, we took  $84 \mu\text{Hz}$  as  $\Delta\nu_{\text{low}}$ , being the most common value obtained by the different diagnostic techniques.

We also searched for a pattern connected to the rotational splitting in the p-mode regime. At first order in the perturbative theory, rotation splits the oscillation modes of the same order and spherical degree in the form:

$$\omega_{nlm} = \omega_{nl} + m\Omega, \quad (6)$$

where  $\omega$  is the angular frequency and  $\Omega$  is the angular rotation frequency. According to Eq. 6, this would allow to find a pattern corresponding to the rotation frequency in the periodogram, i.e., a regular structure related to  $\Omega$ . However, this simple distribution of

frequencies is only valid for slow rotators. Even at moderate rotations (around  $50 \text{ km s}^{-1}$ ) the picture changes: rotational splittings are not symmetric anymore and even the centroid can displace (see e.g. [Suárez et al. 2006](#), for more details). These effects may hamper the identification of a rotational splitting.

Nonetheless, some theoretical works computing 2D non-perturbative models and rapid rotation ([Reese et al. 2017](#)) combined the AC and the FT (following [García Hernández et al. 2009](#)) to search for the patterns of the low-order large frequency separation and the rotational splitting. They pointed out that  $\Delta\nu_{\text{low}}$  or its half value comes up usually clearer in the FT, whereas twice the rotational splitting can be better found with AC diagnostic. Additionally, combinations between the large separation and the rotational splitting have also been proposed ([Evano et al. 2019](#)).

From the observational side, [Paparó et al. \(2016\)](#) searched for regularities in the frequency spectra of  $\delta$  Sct stars stars observed by CoRoT using a visual inspection and by means of a semi-automatic algorithm that look for repeated spacings between frequencies. They claimed that some of these spacings might be not only the large separation but also the rotational splitting or a linear combination of both. The work of [Barceló Forteza et al. \(2017\)](#) used the AC, FDH, ED and TRUFAS ([Régulo & Roca Cortés 2002](#)) to find rotational splittings in a sample of four  $\delta$  Sct stars stars. The main idea was that structural characteristics of the star, such as rotation, alter the power spectra, particularly in the flat plateau, where the density of low amplitude frequencies increases with the rotation rate. [Ramón-Ballesta et al. \(2021\)](#) related regularities found in the frequency spectra of binary  $\delta$  Sct stars to their rotational splitting. In their work, they used the FT, the AC and the FDH as in the present research.

The FDH for two of our stars, TIC 410732825 (Fig. 7) and TIC 285935852 (Fig. A.2) shows a prominent peak around  $9\text{-}11 \mu\text{Hz}$  that could be the identified as the angular rotation. In fact, Fig. 7 also shows significant peaks at both sides of the one identified as  $\Delta\nu_{\text{low}}$ , implying the presence of spacings corresponding to the large separation plus and minus the rotational splitting.

In the case of TIC 285935852, Fig. 8 displays the periodogram with the most visible modes, between  $40$  and  $60 \mu\text{Hz}$ . It is indicated the low-order large separation, with a value of around  $\Delta\nu_{\text{low}} = 7d^{-1}$ . Some other periodicity can also be noticed that might correspond to twice the rotational splitting of around  $0.9\text{-}1.0 d^{-1}$ , or equivalently,  $9\text{-}11 \mu\text{Hz}$ . This is consistent with the results by [Reese et al. \(2017\)](#). The  $m \neq 0$  modes are not symmetric respect to the centroid and the centroid itself is displaced but a spacing corresponding to double the rotational splitting still remains. To check this behaviour, the modes distribution is compared to a compatible model computed with MESA and FILOU codes (see Sec. 6), taking rotation into account and up to second order in the perturbative theory for the adiabatic oscillation computation (including near-degeneracy effects and stellar structure deformation). The mode distribution is similar in both the model and the observation although the limitations of our calculations are clear. Some theoretical modes lie close to the observed ones but others do not. There are missing or completely off modes from the model. Fortunately, the spacings remain although our mode identification might mismatch if we use non-rotating models. We emphasise the importance of taking rotation into account when modelling  $\delta$  Sct stars, as also pointed out by [Murphy et al. \(2021\)](#), although they used non-rotating models to carry out a mode identification.

Another seismic index, recently discovered for  $\delta$  Sct stars, is the frequency at maximum power, related to the effective temperature of the star, as shown in the works of BF2018, BF2020, BK2018 and H2021. This relation is very dependent on the evolutionary stage of the star and dispersion can be very high in some cases. There are



**Figure 5.** The four selected targets in the field of  $\alpha$  Per from which we have obtained regularities in the frequency spectra

**Table 2.** Seismic indices of the selected targets from  $\alpha$  Per to constrain the models, in order to determine its ages

TIC	$\Delta\nu_{\text{low}} (d^{-1})$	$\omega_r (\mu\text{Hz})$	$\nu_{\text{max},0} (\mu\text{Hz})$	$\nu_{\text{max},w} (\mu\text{Hz})$
410732825	$63 \pm 1$	$10 \pm 1$	-	-
354792288	$83 \pm 1$	-	$671 \pm 1$	$623 \pm 1$
285935852	$82 \pm 1$	$10 \pm 1$	$625 \pm 1$	$579 \pm 1$
252829836	$71 \pm 1$	-	-	-

different ways of defining the frequency at maximum power. [Balona & Dziembowski 2011](#) defines it with the peak of maximum amplitude in the spectrum,  $\nu_{\text{max},0}$ . BF2018 do it through the amplitude weighted mean of all the most significant extracted peaks,  $\nu_{\text{max},w}$ , and H2021 through the maximum of the autocorrelation of the power spectra correlation,  $\nu_{\text{max},2D}$ . TIC 354792288 and TIC 285935852 show a set of very grouped frequencies between  $40$  and  $60 \mu\text{Hz}$  (Fig. 9), that seems not to be mixed modes. With both stars, we used  $\nu_{\text{max},0}$  from BF2018 and BK2018,  $\nu_{\text{max},w}$  from BF2020, and  $\nu_{\text{max},2D}$  from H2021, in order to determine effective temperatures.

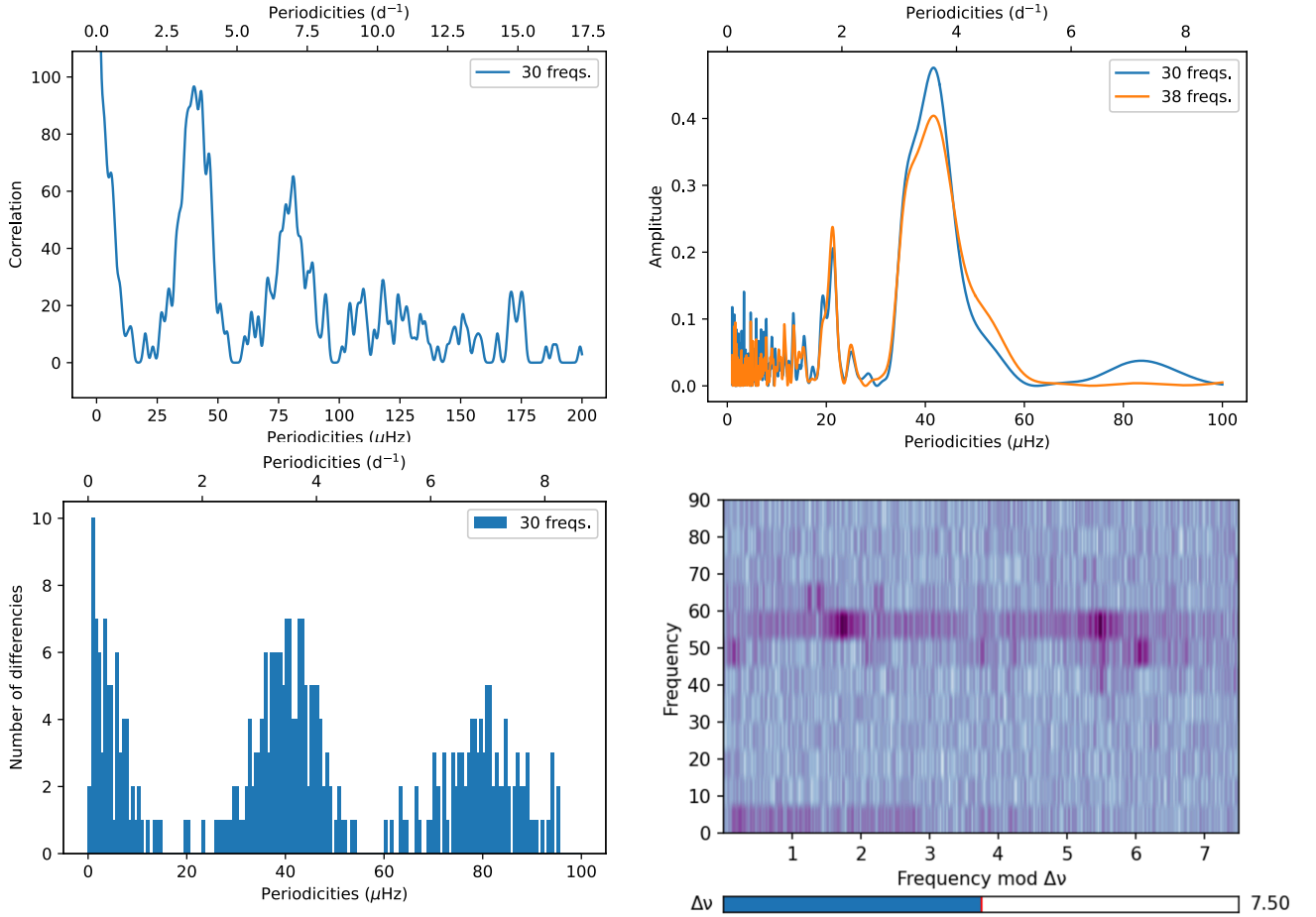
Tab. 2 shows the values of all these seismic indices that will serve to constrain the models to try to date these four stars accurately, and therefore, the cluster.

## 6 The grid of models

For this purpose, we have built a grid of 1D stellar models using the code MESA for the evolution, and FILOU for calculating the oscillation modes, because it takes into account the stellar distortion due to the centrifugal force in the oscillation frequency computation, and rotation up to second order in the perturbation approximation, including near-degeneracy effects. We have taken orders  $2 \leq n \leq 8$  and degrees  $0 \leq l \leq 2$ , for calculating the low-order large separation of the models ([Suárez et al. 2014](#); [Rodríguez-Martín et al. 2020](#)).

Regarding the evolutionary models, we built a grid using the parameters of Tab. 3, delimiting ages between  $20$  and  $200$  Myrs. According to  $Z_{\text{base}}$  values, we selected the Type2 tables for opacities.

‡ [Echelle 1.5.1](#)



**Figure 6.** Measured regularities in the frequency spectra of TIC 354792288. The top left panel represents the autocorrelation diagram (AC) of periodicities calculated with the first 30 frequencies extracted by MM, showing peaks around  $42 \mu\text{Hz}$  and  $84 \mu\text{Hz}$ . Top right, the Fourier transform (FT) for the periodicities of those first 30 frequencies, showing a peak around  $41 \mu\text{Hz}$ . Bottom left, the frequency difference histogram (FDH) with the first 30 frequencies, showing peaks around  $41 \mu\text{Hz}$  and  $82 \mu\text{Hz}$ . Bottom right, the echelle diagram (ED) showing a vertical ridge when  $\Delta\nu_{\text{low}}$  is chosen around  $7.5 d^{-1}$ , equivalent to  $86 \mu\text{Hz}$ . ED is done with Python package Echelle 1.5.1, developed by [Hey & Ball 2020](#)‡

**Table 3.** Selected values for the parameters of the grid of models built with MESA

Parameter	Range	Step
Age	[20, 200] Myr	1 Myr
$M (M_{\odot})$	[1.6, 2.6]	$0.1 M_{\odot}$
$Z_0$	[0.010, 0.020]	0.002
$\Omega/\Omega_c$	[0.10, 0.25]	0.05
$\alpha$	2.0	Fixed

For the mixing length parameter we considered  $\alpha = 2.0$ , and a diffusion coefficient for mixing of elements of  $D_{\text{mix}} = 1/30$  ([Heger et al. 2000](#)). Overshooting was not considered. In total, 24965 models were calculated.

## 7 Seismic determination of the age of $\alpha$ Per

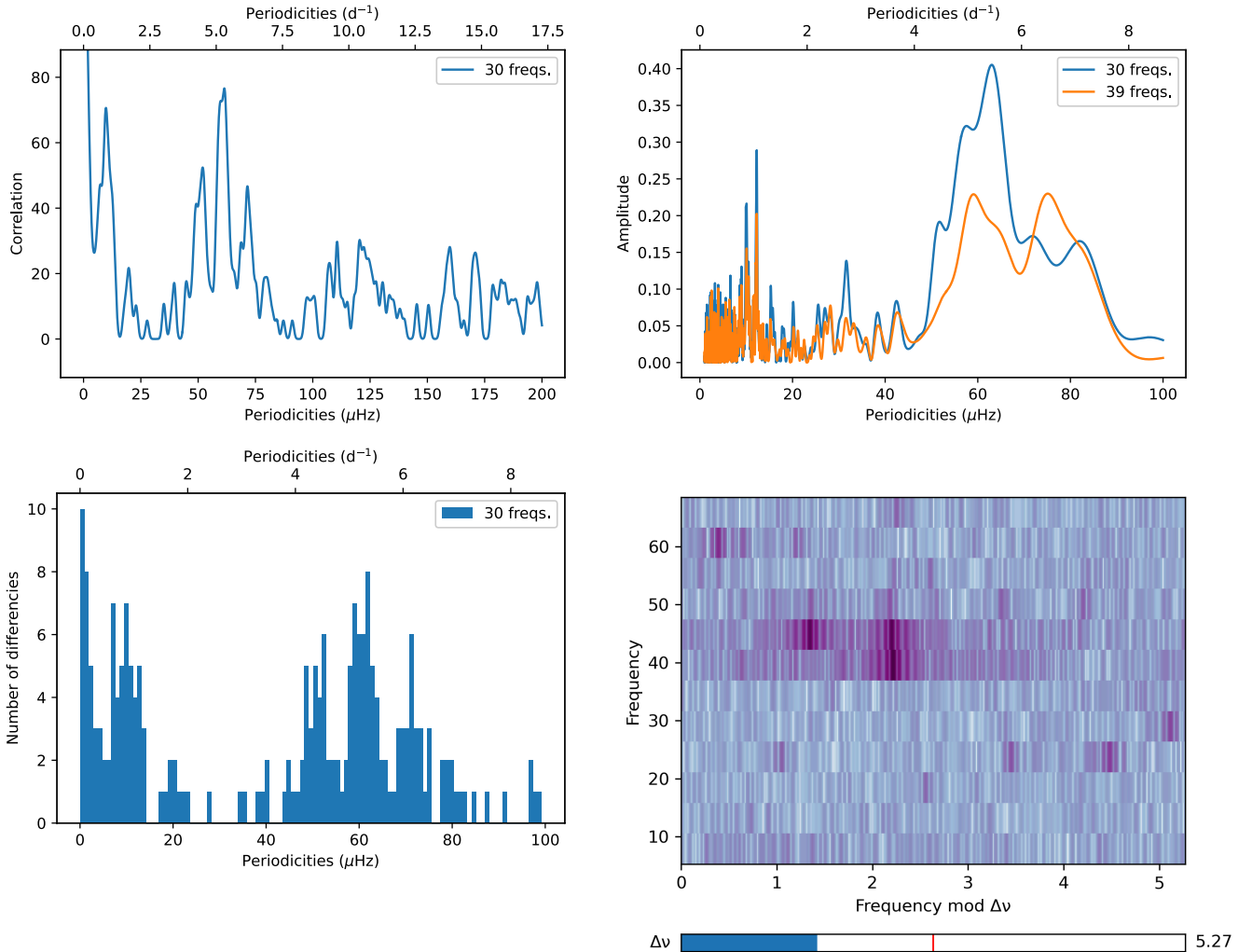
Fig. 10 shows the effective temperature vs age plots for all the calculated models, using the relationships between the frequency at maximum power and the effective temperature presented in Sec. 5,

and without using any of them. The constrained models for TIC 410732825 are represented in green, for TIC 354792288 in blue, for TIC 285935852 in red and for TIC 252829836 in orange. If we do not use any kind of relationship between the frequency at maximum power and the effective temperature to constrain the ages of our four stars (top left panel), then the age of the cluster would be between 103 and 160 Myr. If we use the  $\nu_{\text{max},w}\text{-}T_{\text{eff}}$  relationship from BF2020 (middle right panel) or  $\nu_{\text{max},2D}\text{-}T_{\text{eff}}$  from H2021 (bottom left panel), then the constrained models for our four stars do not show common ages. However, in the case of BF2020, the possible ages for TIC 410732825 and TIC 354792288 are separated by a gap of just 2-3 million years, which is too narrow to rule out the possibility that they may overlap around a common age of about 100 Myr.

And finally, if we use the  $\nu_{\text{max}0}\text{-}T_{\text{eff}}$  relationship from BF2018 (top right panel) and BK2018 (middle left panel), the result is that  $\alpha$  Per could have an age between 103 and 131 Myr.

Tab. 4 shows the observed values of the main parameters of these four stars, and Tab. 5 the values corresponding to the constrained models when using relations BF2018 or BK2018.

Regarding TIC 410732825, the models tell us that it has a larger radius and a lower density than the calculated from the observed



**Figure 7.** Measured regularities in the frequency spectra of TIC 410732825. The autocorrelation diagram (AC) (top left), the Fourier transform (FT) (top right), and the frequency difference histogram (FDH) (bottom left), calculated with the first 30 frequencies extracted by MM, show a prominent peak around  $62 \mu\text{Hz}$ . And also, the echelle diagram (ED) (bottom right) show a very clear vertical ridge when  $\Delta\nu_{\text{low}}$  is around  $5.3 d^{-1}$ , equivalent to  $61 \mu\text{Hz}$ .

parameters. The density obtained from the large separation using the [García Hernández et al. \(2017\)](#) relationship seems to corroborate this. The values of the projected rotation of [Kounkel et al. \(2019\)](#) and the seismic rotation are very similar, so the star seems to be equator on. This would explain why the possible values of the observed luminosity are below those of the models. If the observed parameters for TIC 252829836 were correct, then the star would have an age of hundreds of millions of years in order to be compatible with the measured large separation. This solution was therefore discarded, and points toward a wrong estimate of its physical parameters.

Our four stars, that seem to be members of the cluster, according to [Lodieu et al. \(2019\)](#), and not seem to be binary systems, according to [Kounkel et al. \(2019\)](#), point to an older age for the cluster of at least 103 Myr.

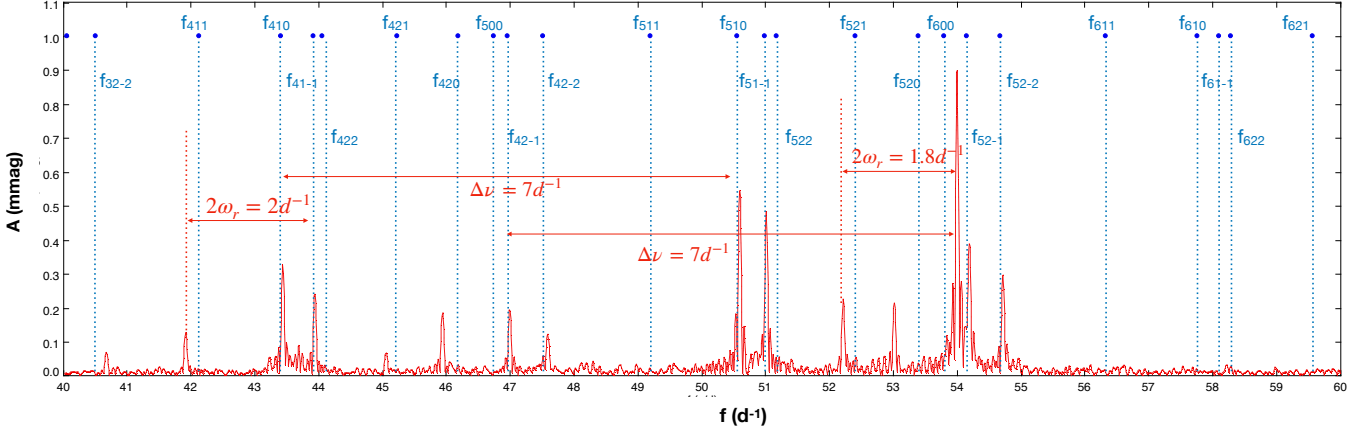
## 8 Conclusions

We have tested an asteroseismic method in order to determine the age of the young open cluster  $\alpha$  Per. The technique is based on

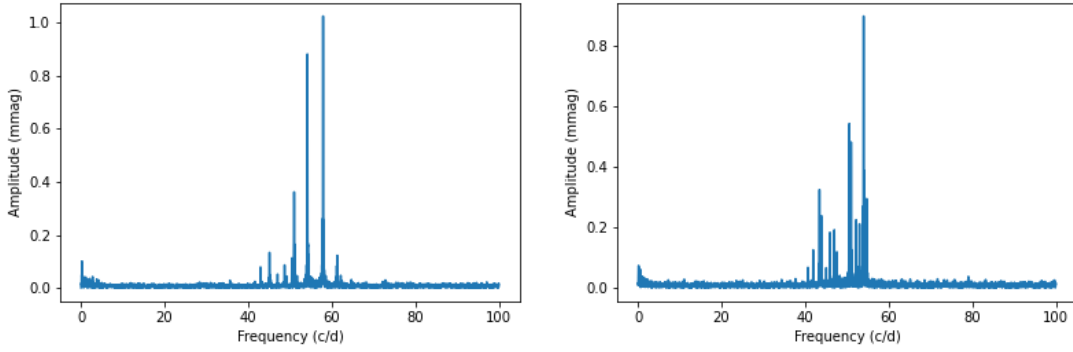
finding seismic indices, such as the low-order large separation, the frequency at maximum power and the rotation frequency in a sample of 11  $\delta$  Sct stars belonging to the field of the cluster. With them we have used the FT, the AC, the FDH and the ED. In four of them, TIC 410732825, TIC 354792288, TIC 285935852 and TIC 252829836 we have measured the low-order regime large separation. With TIC 410732825 and TIC 285935852 we have found evidences of the angular rotation frequency. With the necessary caution, considering that they are ZAMS stars, we have used the relations between the frequency at maximum power and the effective temperature to constrain the age of  $\alpha$  Per between 103 and 131 Myr.

There are more sources of uncertainty using the isochrone fitting method than the one we propose here based only on seismic indices, and with no need of doing frequency mode identification. The grid we have used can be extended to take into account the variability of parameters such as overshooting, the mixing length parameter or the diffusion of angular.

Notice that this asteroseismic approach based on seismic indices is more reliable than any method based only upon isochrone fitting



**Figure 8.** Periodogram of TIC 285935852, showing a measured low-order large separation of around  $\Delta\nu_{\text{low}} = 7d^{-1}$ , and a rotational splitting of around  $\omega_r = 1.9d^{-1}$ . The modes distribution is compared to a suitable model calculated with MESA-FILOU (see Sec. 6). Some modes seem to follow the frequency distribution of the model. Others, related to rotation, where the frequency splitting is visible, don't do it. Even the centroids are displaced from the model



**Figure 9.** Periodograms for TIC 354792288 and TIC 285935852, showing very grouped frequencies between 40 and 60  $\mu\text{Hz}$

in its numerous uncertainties. The estimated age for  $\alpha$  Per must be considered with caution until new studies with more  $\delta$  Sct stars and/or a more detailed modeling allow us to confirm it. In any case, the method based on seismic indices looks a promising tool for dating young stellar clusters. For the frequency analysis, we have developed a new code, MULTIMODES, MM (MultiModes), that we present in this paper. The code shows a good reliability after testing with synthetic light curves, and after being compared with one of the most reliable codes in the field, SIGSPEC. Moreover, MULTIMODES is more versatile in terms of frequency analysis control. It has allowed us to find nine previously unknown  $\delta$  Sct stars from the original sample of 32 analysed stars.

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### Data Availability

Tables of the most significant peaks, corresponding to each of the 11  $\delta$  Scuti stars of our sample, extracted with MM, are available in [VizieR DataBase](#). In them we can find the values of the frequencies, amplitudes, phases, corresponding errors and S/N of the extracted peaks.

**Table 4.** Parameters of the selected targets. References: <sup>1</sup>Stassun et al. 2019, <sup>2</sup>Kounkel et al. 2019, <sup>3</sup>García Hernández et al. 2017, <sup>4</sup>Barceló Forteza et al., <sup>5</sup>Bowman & Kurtz 2018, <sup>6</sup>Barceló Forteza et al. 2020, <sup>7</sup>Hasanzadeh et al. 2021

TIC	$M(M_{\odot})^1$	$R(R_{\odot})^1$	$\bar{\rho}(\bar{\rho}_{\odot})$	$\log g^1$	$T_{\text{eff}}(K)^1$	$\log(L/L_{\odot})^1$	$v \sin i (kms^{-1})^2$	$\Delta v_{\text{low}}(cd^{-1})$	$\bar{\rho}_{\Delta v_{\text{low}}}(\bar{\rho}_{\odot})^3$	$T_{v_{\text{max},\delta}}(K)^4$	$T_{v_{\text{max},\delta}}(K)^5$	$T_{v_{\text{max},w}}(K)^6$	$T_{v_{\text{max},2D}}(K)^7$
410732825	[1.983:2.651]	[1.592:1.686]	[0.41:0.66]	[4.30:4.45]	[8851:9453]	[1.18:1.28]	[71:107]	[62:64]	[0.31:0.33]	-	-	-	-
354792288	[1.666:2.370]	[1.518:1.624]	[0.39:0.68]	[4.27:4.44]	[7849:8605]	[0.93:1.08]	[107:119]	[82:84]	[0.54:0.57]	[8202:9226]	[7883:8389]	[8351:8931]	[8255:8536]
285935852	[1.465:2.063]	[1.545:1.645]	[0.33:0.56]	[4.20:4.36]	[7363:7815]	[0.84:0.92]	[68:73]	[81:83]	[0.53:0.56]	[8099:9109]	[7808:8282]	[8202:8772]	[8183:8455]
252829836	[1.313:1.863]	[1.617:1.747]	[0.25:0.44]	[4.10:4.27]	[6994:7272]	[0.80:0.84]	[37:39]	[70:72]	[0.39:0.42]	-	-	-	-

**Table 5.** Constrained parameters of the models corresponding to our selected targets, using the relations BF2018 or BK2018

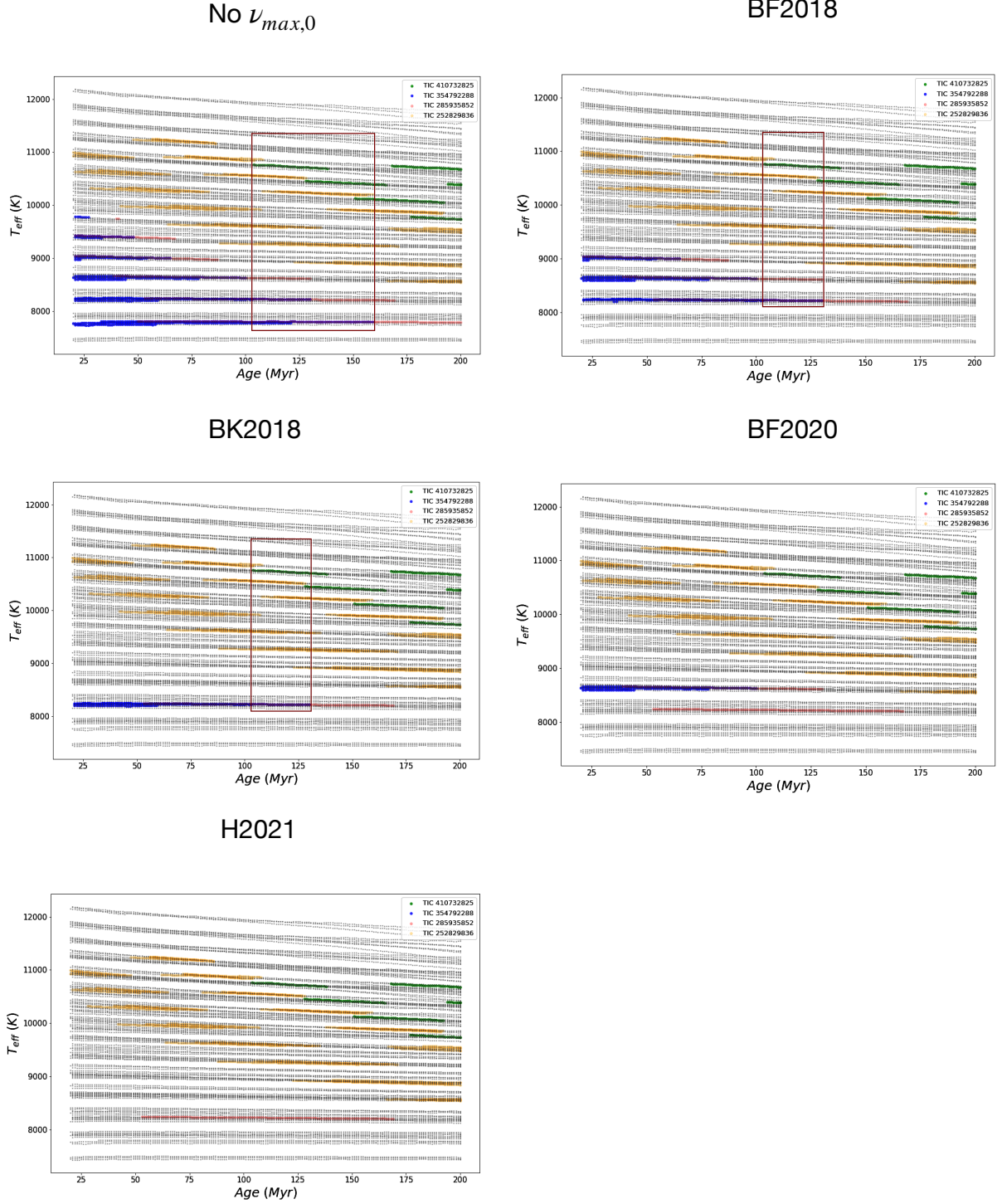
TIC	$M(M_{\odot})$	$R(R_{\odot})$	$\bar{\rho}(\bar{\rho}_{\odot})$	$\log g$	$T_{\text{eff}}(K)$	$\log(L/L_{\odot})$	$Z_0$	$v(kms^{-1})$	$\Delta v_{\text{low}}(cd^{-1})$	Age (Myr)
410732825	[2.3:2.6]	[1.95:2.09]	[0.28:0.31]	[4.20:4.24]	[9731:10759]	[1.49:1.70]	[0.016:0.020]	[96:99]	[62:64]	[104:200]
354792288	[1.65:1.75]	[1.48:1.50]	[0.50:0.52]	[4.31:4.33]	[8192:8245]	[0.95:0.96]	[0.016:0.020]	[70:114]	[82:84]	[21:130]
285935852	[1.65:1.75]	[1.48:1.52]	[0.48:0.52]	[4.31:4.33]	[8202:8241]	[0.95:0.97]	[0.016:0.020]	[69:70]	[81:83]	[53:169]
252829836	[1.9:2.6]	[1.68:1.94]	[0.36:0.40]	[4.24:4.30]	[8541:11248]	[1.13:1.71]	[0.016:0.020]	[69:126]	[70:72]	[20:200]

## REFERENCES

- Andrae R., et al., 2018, *A&A*, **616**, A8
- Baglin A., Michel E., Auvergne M., COROT Team 2006, in Proceedings of SOHO 18/GONG 2006/HELAS I, Beyond the spherical Sun. p. 34
- Balona L. A., 2014, *MNRAS*, **439**, 3453
- Balona L. A., Dziembowski W. A., 2011, *MNRAS*, **417**, 591
- Barceló Forteza S., Roca Cortés T., García Hernández A., García R. A., 2017, *A&A*, **601**, A57
- Barceló Forteza S., Roca Cortés T., García R. A., 2018, *A&A*, **614**, A46
- Barceló Forteza S., Moya A., Barrado D., Solano E., Martín-Ruiz S., Suárez J. C., García Hernández A., 2020, *A&A*, **638**, A59
- Basri G., Martín E. L., 1999, *ApJ*, **510**, 266
- Bedding T. R., et al., 2020, *Nature*, 581, 147
- Bowman D. M., Kurtz D. W., 2018, *MNRAS*, **476**, 3169
- Choi J., Dotter A., Conroy C., Cantiello M., Paxton B., Johnson B. D., 2016, *ApJ*, **823**, 102
- Dotter A., 2016, *ApJS*, **222**, 8
- Evano B., Lignières F., Georgeot B., 2019, *Astronomy & Astrophysics*, **631**, A140
- Gaia Collaboration et al., 2017, VizieR Online Data Catalog, pp J/A+A/601/A19
- Gaia Collaboration et al., 2018, *A&A*, **616**, A10
- García Hernández A., et al., 2009, *A&A*, **506**, 79
- García Hernández A., Martín-Ruiz S., Monteiro M. J. P. F. G., Suárez J. C., Reese D. R., Pascual-Granado J., Garrido R., 2015, *ApJ*, **811**, L29
- García Hernández A., et al., 2017, *MNRAS*, **471**, L140
- Girardi L., Bressan A., Bertelli G., Chiosi C., 2000, *A&AS*, **141**, 371
- Grigahcène A., et al., 2010, *ApJ*, **713**, L192
- Hasanzadeh A., Safari H., Ghasemi H., 2021, *MNRAS*, **505**, 1476
- Heger A., Woosley S. E., Langer N., 2000, *New Astron. Rev.*, **44**, 297
- Hey D., Ball W., 2020, Echelle: Dynamic echelle diagrams for asteroseismology, doi:10.5281/zenodo.3629933
- Koch D. G., et al., 2010, *ApJ*, **713**, L79
- Kounkel M., et al., 2019, *The Astronomical Journal*, **157**, 196
- Lodieu N., Pérez-Garrido A., Smart R. L., Silvotti R., 2019, *A&A*, **628**, A66
- Makarov V. V., 2006, *AJ*, **131**, 2967
- Martín E. L., Dahm S., Pavlenko Y., 2001, in von Hippel T., Simpson C., Manset N., eds, *Astronomical Society of the Pacific Conference Series* Vol. 245, *Astrophysical Ages and Times Scales*. p. 349
- Murphy S. J., Hey D., Van Reeth T., Bedding T. R., 2019, *Monthly Notices of the Royal Astronomical Society*, **485**, 2380
- Murphy S. J., Bedding T. R., White T. R., Li Y., Hey D., Reese D., Joyce M., 2021, arXiv e-prints, p. arXiv:2111.04203
- Netopil M., Paunzen E., 2013, *A&A*, **557**, A10
- Paparo M., Benkó J. M., Hareter M., Guzik J. A., 2016, *ApJ*, **822**, 100
- Pascual-Granado J., Garrido R., Suárez J. C., 2015, *A&A*, **575**, A78
- Pascual-Granado J., Suárez J. C., Garrido R., Moya A., Hernández A. G., Rodón J. R., Lares-Martiz M., 2018, *Astronomy and Astrophysics*, **614**, 1
- Paxton B., 2019, Modules for Experiments in Stellar Astrophysics (MESA), doi:10.5281/zenodo.3473377
- Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F., 2011, *ApJS*, **192**, 3
- Paxton B., et al., 2013, *ApJS*, **208**, 4
- Paxton B., et al., 2015, *ApJS*, **220**, 15
- Pietrinferni A., Cassisi S., Salaris M., Castelli F., 2004, *ApJ*, **612**, 168
- Press W. H., Rybicki G. B., 1989, *ApJ*, **338**, 277
- Prosser C. F., 1992, *AJ*, **103**, 488
- Ramón-Ballesta A., García Hernández A., Suárez J. C., Rodón J. R., Pascual-Granado J., Garrido R., 2021, *MNRAS*, **505**, 6217
- Reegen P., 2007, *Astronomy & Astrophysics*, **467**, 1353
- Reese D. R., Lignières F., Ballot J., Dupret M.-A., Barban C., van 't Veer-Menneret C., MacGregor K. B., 2017, *Astronomy & Astrophysics*, **601**, A130
- Régulo C., Roca Cortés T., 2002, *A&A*, **396**, 745
- Ricker G. R., et al., 2014, in Proc. SPIE. p. 914320 (arXiv:1406.0151), doi:10.1117/12.2063489
- Rodríguez-Martín J. E., García Hernández A., Suárez J. C., Rodón J. R., 2020, *MNRAS*, **498**, 1700
- Scargle J. D., 1982, *ApJ*, **263**, 835
- Silaj J., Landstreet J. D., 2014, *A&A*, **566**, A132
- Stassun K. G., et al., 2019, *The Astronomical Journal*, **158**, 138
- Stauffer J. R., et al., 1999, *ApJ*, **527**, 219
- Suárez J. C., Goupil M. J., 2008, *Ap&SS*, **316**, 155
- Suárez J. C., Michel E., Pérez Hernández F., Lebreton Y., Li Z. P., Fox Machado L., 2002, *A&A*, **390**, 523
- Suárez J. C., Goupil M. J., Morel P., 2006, *Astronomy & Astrophysics*, **449**, 673
- Suárez J. C., García Hernández A., Moya A., Rodrigo C., Solano E., Garrido R., Rodón J. R., 2014, *A&A*, **563**, A7
- Uytterhoeven K., et al., 2011, *A&A*, **534**, A125
- VanderPlas J. T., 2018, *ApJS*, **236**, 16

## A Comparative analysis of efficiency between MM and SS

Fig. A.1 represents the executing time of both codes using the light curves of 11  $\delta$  Sct stars stars from our sample. MM is a factor two faster than SS in the cases where the number of extracted peaks is above 200 approximately.



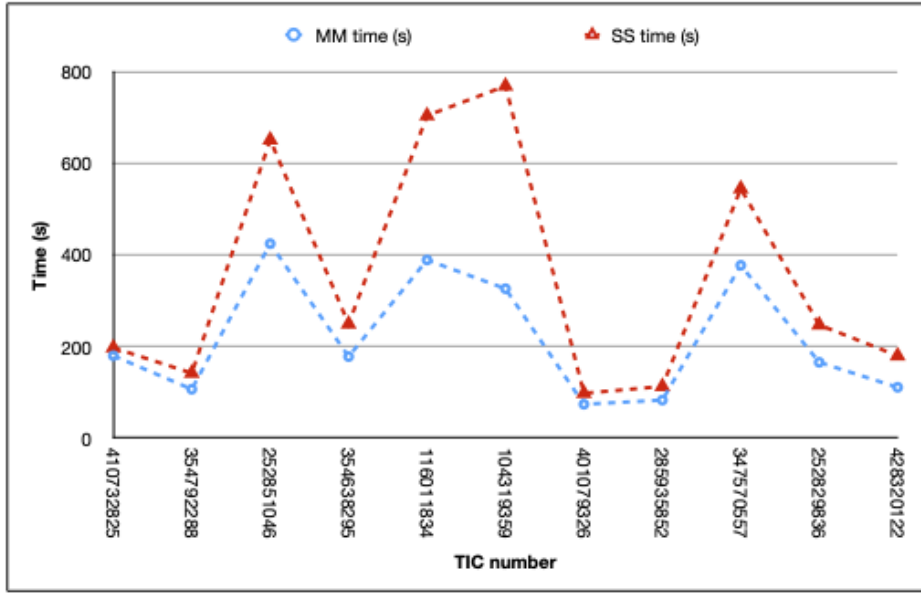
**Figure 10.** Constrained models for dating ages of the four stars, without using the relation between the frequency at maximum power and effective temperature (No  $\nu_{\text{max}}$ ), and using the different relations of BF2018, BF2020, BK2018 and H2021

## B Diagrams of regularities of TIC 285935852 and TIC 252829836

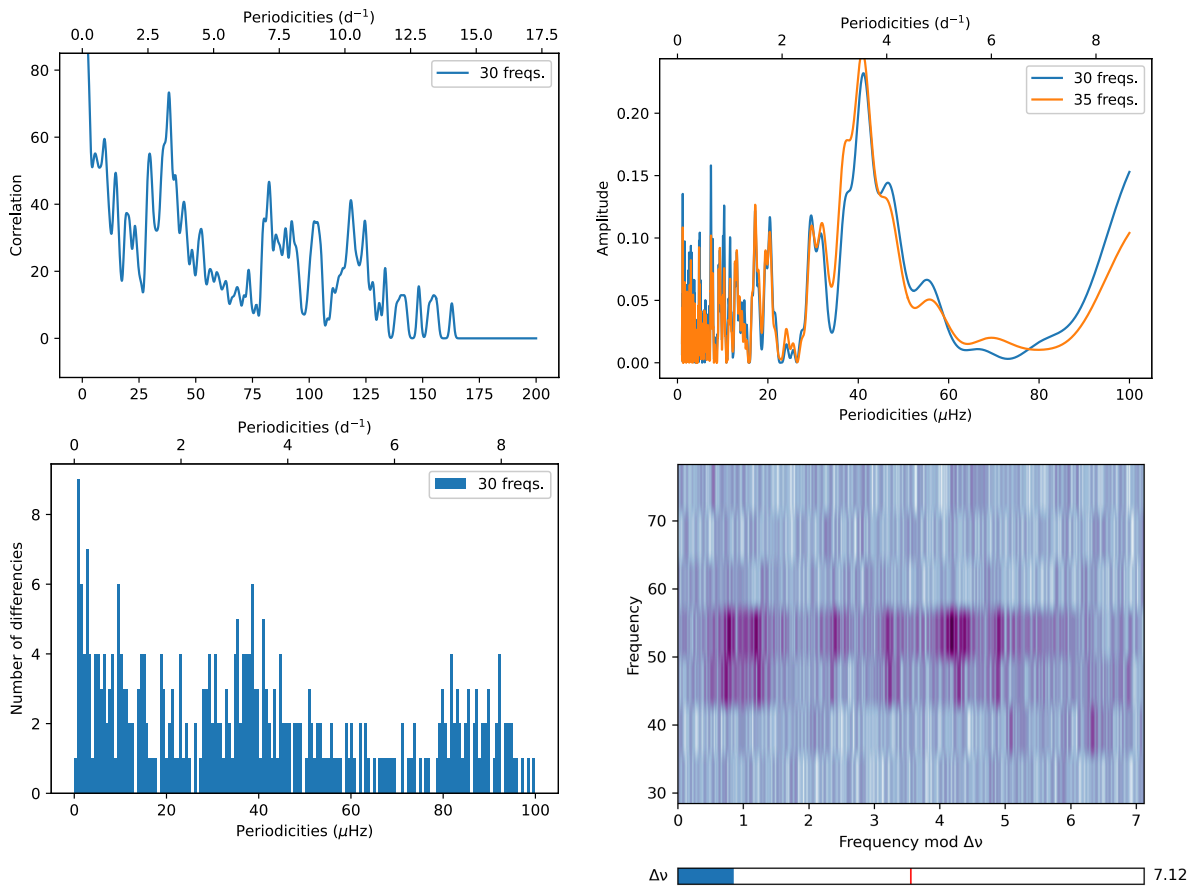
We include here the AC, FT, FDH and ED of TIC 285935852 (Fig. A.2) and TIC 252829836 (Fig. A.3), which together with TIC 410732825 (Fig. 7) and TIC 354792288 (Fig. 6), complete the four

stars analyzed for which regularities have been found in their corresponding frequency spectra.

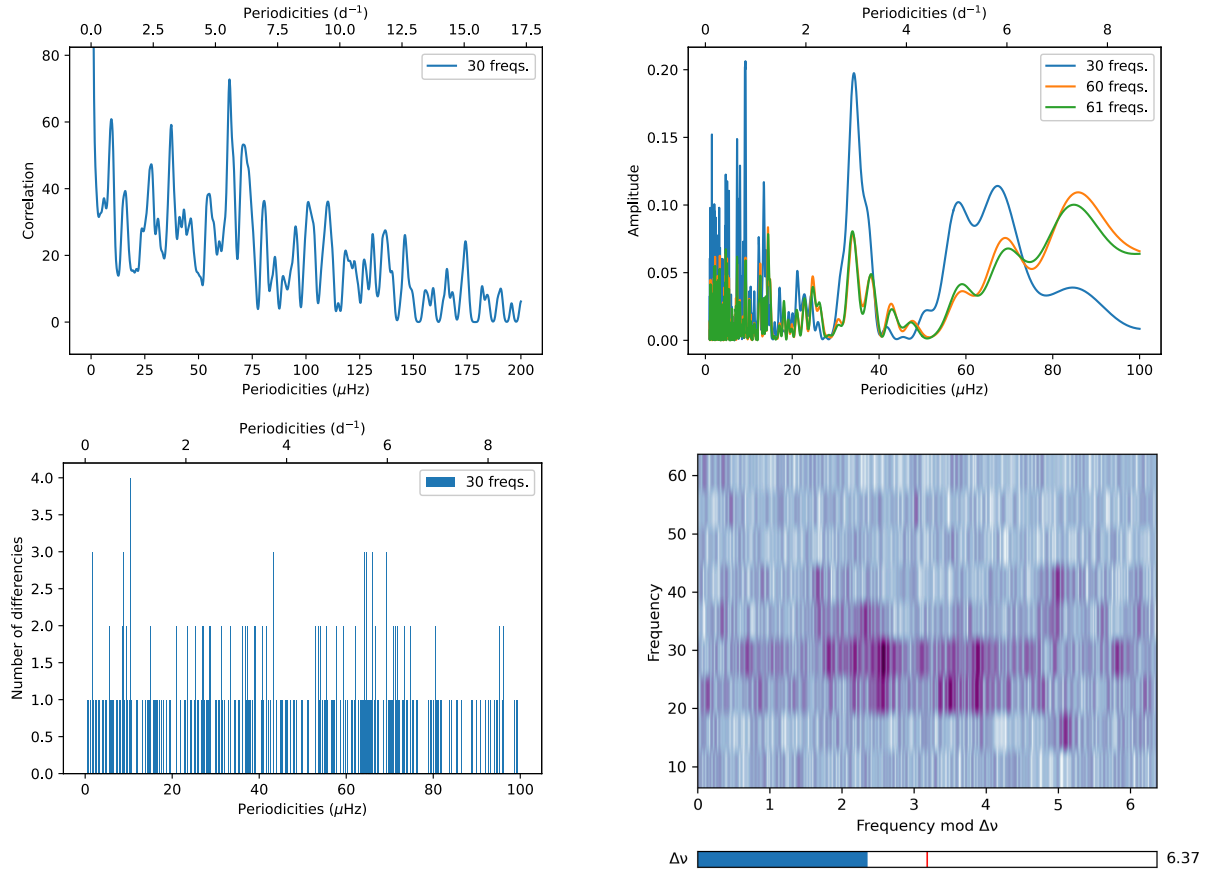
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**Figure A.1.** Comparative analysis of efficiency between MM and SS, in terms of computing time, done with the sample of 11  $\delta$  Sct stars



**Figure A.2.** Measured regularities for TIC 285935852



**Figure A.3.** Measured regularities for TIC 252829836